GRAVITATION

1. INTRODUCTION

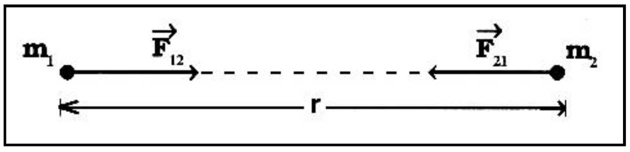
The constitutents of the universe are galaxy, stars, planets, comets, asteriods, meteroids. The force which keeps them bound together is called gravitational force. Gravitation is a nature phenomenon by which material objects attract towards one another.

In 1687 A.D. English Physicist, Sir Isaac Netwon published principia Mathematica, which explains the inverse-square law of gravitation.

2. NEWTON'S LAW OF GRAVITATION

2.1 Defintion

Every particle of matter attracts every other particle of matter with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



2.2 Mathematical Form

If m_1 and m_2 are the masses of the particles and r is the distance between them, the force of attraction F between the particles is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore \quad F = G\frac{m_1 m_2}{r^2}$$

Where G is the universal constant of gravitation.

2.3 Vector Form

In vector form, Newton's law of gravitation is represented in the following manner. The force $(\vec{F}_{21})$ exerted on particle m_2 by particle m_1 is given by,

$$\vec{F}_{21} = -G\frac{m_1 m_2}{r^2}(\hat{r}_{12}) \qquad ...(i)$$

Where $(\hat{r}_{12})$ is a unit vector drawn from particle m_1 to particle m_2.

Similarly, the force $(\vec{F}_{12})$ exerted on particle m_1 by particle m_2 is given by

$$\vec{F}_{12} = +G\frac{m_1 m_2}{r^2}(\hat{r}_{12}) \qquad ...(ii)$$

Where $(\hat{r}_{12})$ is a unit vector drawn from particle m_1 to particle m_2

From (i) and (ii)

$$\therefore \quad \vec{F}_{12} = -\vec{F}_{21}$$

3. UNIVERSAL CONSTANT OF GRAVITATION

Universal gravitation constant is given as, $G = \frac{Fr^2}{m_1 m_2}$

Suppose that, $m_1 = m_2 = 1$, and $r = 1$ then $G = F$

∴ Universal gravitation constant is numerically equal to the force of attraction between two unit masses placed at unit distance apart.

3.1 Unit

$$\text{SI unit}: \frac{\text{newton}(\text{metre})^2}{(\text{kilogram})^2} = \frac{Nm^2}{kg^2}$$

CGS Unit : dyne cm^2/gm^2

3.2 Value of G

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

Dimensions of G

$$[G] = \frac{[F][r^2]}{[m_1 m_2]} = \frac{[M^1L^1T^2][M^0L^2T^0]}{[M^2L^0T^0]}$$

$$= [M^{-1}L^3T^{-2}]$$

Note...

1. The gravitational force is independent of the intervening medium.
2. The gravtional force is a conservative force.
3. The force exerted by the first particle on the second is exactly equal and opposite to the force exerted by the second particle on the first.
4. The gravitational force between two particles act along the line joining the two particles and they from an action-reaction pair.

4. VARIATION IN 'g'

4.1 The Acceleration due to Gravity at a height h above the Earth's surface

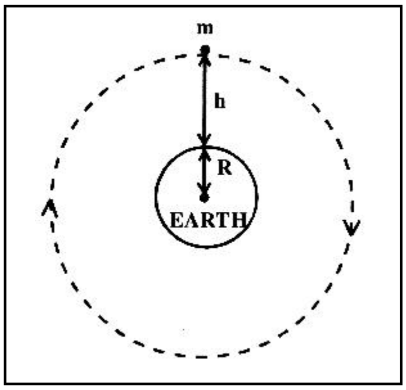
Let M and R be the mass and radius of the earth and g be the acceleratio due to gravity at the earth's surface. Suppose that a body of mass m is placed on the surface of the earth.

The weight 'mg' of the body is equal to the gravitational force acting on it.

$$\therefore \quad mg = \frac{GMm}{R^2}$$

$$\therefore \quad g = \frac{GM}{R^2} \qquad \text{...(i)}$$

Now suppose that the body is raised to a height h, above the earth's surface, the weight of the body is now mg_h and the gravitational force acting on it is $\frac{GMm}{(R+h)^2}$



$$\therefore \quad mg_h = \frac{GMm}{(R+h)^2}$$

$$\therefore \quad g_h = \frac{GM}{(R+h)^2} \qquad \text{...(ii)}$$

Dividing eq (ii) by eq (i), we get,

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore \quad g_h = \left[\frac{R^2}{(R+h)^2}\right] g$$

4.2 Acceleration due to gravity at a very small height

$$g_h = g\left(\frac{R+h}{R}\right)^2$$

$$= g\left(1-\frac{h}{R}\right)^{-2}$$

$$= g\left(1-\frac{2h}{R}+\frac{h^2}{R^2}........\right)$$

If h << R, then neglecting high power's of 'h' we get,

$$g_h = g\left(1-\frac{2h}{R}\right)$$

4.3 Effect of depth on a acceleration due to Gravity

Also g in terms of ρ

$$g = \frac{GM}{R^2}$$

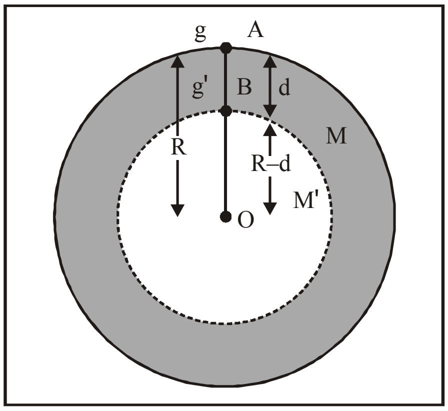
If ρ is density of the material of earth, then

$$M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore \quad g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$\therefore \quad g = \frac{4}{3}\pi G R \rho \qquad \text{...(i)}$$

Let g_d be acceleration due to gravity at the point B at a depth x below the surface of earth. A body at the point B will experience force only due to the portion of the earth of radius OB (R – d). The outer spherical shell, whose thickness is d, will not exert any force on body at point B. Because it will acts as a shell and point is inside.



Now, $M' = \frac{4}{3}\pi(R - x)^3 \rho$

or $\quad g_d = \frac{4}{3}\pi G(R - d)\rho \qquad$ ...(ii)

Dividing the equation (ii) by (i), we have

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G R \rho} = \frac{R-d}{R} \quad \text{or} \quad g_d = g\left(1 - \frac{d}{R}\right) \qquad \text{...(iii)}$$

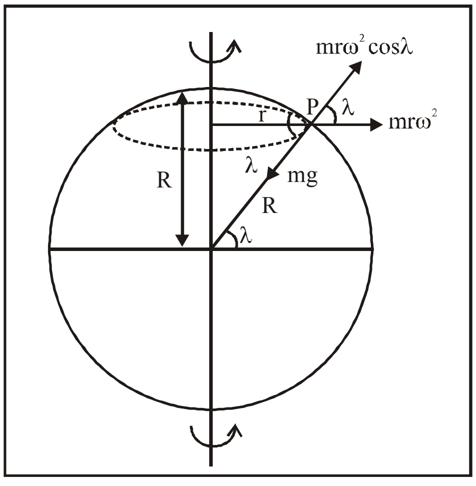
Therefore, the value of acceleration due to gravity decreases with depth.

4.4 Variation of 'g' with latitude due to Rotational motion of Earth

Due to the rotational of the earth the force $mr\omega^2 \cos\lambda$ acts radially outwards. Hence the net force of attraction exerted by the earth of the particle and directed towards the centre of the earth is given by

$mg' = mg - mr\omega^2\cos\lambda$

where g' is the value of the acceleration due to gravity at the point P.



$\therefore \quad g' = g - r\omega^2\cos\lambda$

Now, $r = R\cos\lambda$ (where R is the radius of the earth)

Then $\quad g' = g - (R\cos\lambda)\,\omega^2\cos\lambda$

$\therefore \quad g' = g - R\omega^2\cos^2\lambda$

The effective acceleration due to gravity at a point 'P' is given by,

$g' = g - R\omega^2\cos^2\lambda.$

Thus value of 'g' changes with 'λ' and 'ω'

1. At poles,

$\lambda = 90,$

$g' = g - R\,\omega^2\cos^2 90.$

$g' = g$

This is maximum acceleration due to gravity.

2. At equator

$\lambda = 0,$

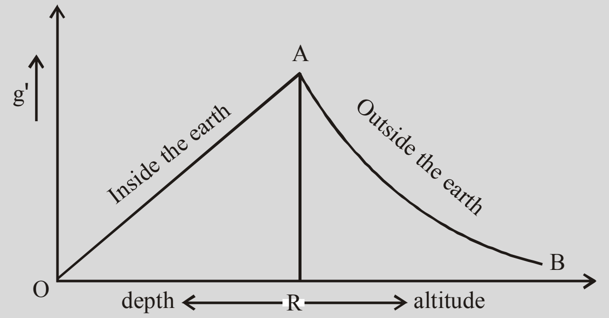
$g' = g - R\omega^2\cos^2 0$

$g' = g - R\omega^2$

This is minimum acceleration due to gravity.

Note...

The variation of acceleration due to gravity according to the depth and the height from the earth's surface can be expressed with help of following graph.



5. SATELLITE

5.1 Definition

Any smaller body which revolves around another larer body under the influence of its gravitation is called a **satellite.** The satellite may be natural or artificial.

1. The moon which revolves around the earth, is a satellite of the earth. There are sixteen satellites revolving around the planet Jupiter. These satellite are called natural satellites.

2. A satellite made and launched into circular orbit by man is called an artificial satellite. The first satellite was launched by USSR named SPUTNIK–I and the first Indian satellite was 'ARYABHATTA'.

5.2 Minimum two stage rocket is used to project a satellite in a cirular orbit round a planet

Suppose that a single stage launching system (i.e. a rocket), carrying satellite at its tip, is used to project the satellite from the surface of the earth. When the fuel in the rocket is ignited, the rocket begins to move upwards. The rocket attains maximum velocity when all the fule is exhausted.

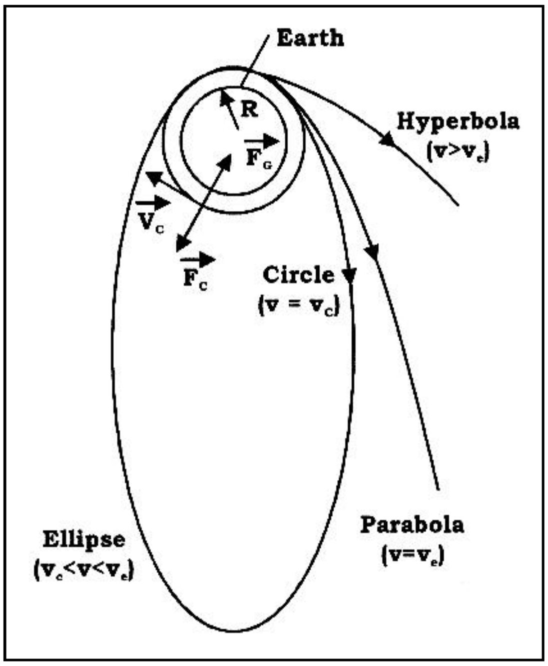
1. If the maximum velocity attained by the rocket is equal to or greater than the escape velocity, the rocket overcomes the eath's gravitational influence and escapes into space alogn with the satellite.

3. If the maximum velocity attained by the rocket is less escape velocity, the rocket cannot overcome the earth's gravitational influence and both the rocket and the satellite eventually fall on the earth's surface due to gravity.

Thus a single stage rocket is unable to launch a satellite in a circular orbit round the earth. Therefore a launching system (i.e. a rocket) having two or more stages must be used to launch a satellite in a circular orbit round the earth.

5.3 Different cases of Projection

When a satellite is taken to some height above the earth and then projected in the horizontal direction, the following four cases may occur, depending upon the magnitude of the horizontal velocity.



1. If the velocity of the projection is less than the critical velocity then the satellite moves in elliptical orbit, but the point of projection is apogee and int he orbit, the satellite comes closer to the earth with its perigee point lying at 180°. If it enters the atmosphere while coming towards perigee it will loose energy and spirally comes down. If it does not enters the atmosphere it will contiune to move in elliptical orbit.

2. If the velocity of the projection is equal to the critical velocity then the satellite moves in circular orbit round the earth.

3. If the velocity of the projection is greater than the critical velocity but less than the escape velocity, then the satellite moves in elliptical orbit and its apoagee, or point of greatest distance from the earth, will be greater than projection height.

4. If the velocity of the projection is equals to the escape velocity, then the satellite moves in parabolic path.

5. It the velocity of the projection is greater than the escape velocity, then orbit will hyperbolic and will escape the gravitational pull of the earth and continue to travel infinitely.

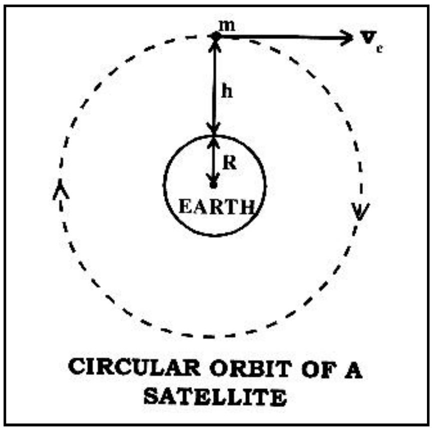
6. ORBITAL VELOCITY

6.1 Definition

The horizontal velocity with which a satellite must be projected from a point above the earth's surface, so that it revolves in a circular orbit round the earth, is called the **orbital velocity** of the satellite.

6.2 An Expression for the Critical Velocity of a Satellite revolving round the Earth

Suppose that a satellite of mass m is raised to a height h above the earth's surface and then projected in a horizontal direction with the orbital velocity v_c. The satellite begins to move round the earth in a circular orbit of radius, R + h, where R is the radius of the earth.



The gravitational force acting on the satellite is $\frac{GMm}{(R+h)^2}$,

where M is the mass of the earth and G is the constant of gravitation.

For circular motion,

Centrifugal force = Centripetal force

$$\therefore \quad \frac{mv_c^2}{(R+h)} = \frac{GMm}{(R+h)^2},$$

$$\therefore \quad v_c = \sqrt{\frac{GM}{(R+h)}}$$

This expression gives the critical velocity of the satellite.

From the expression, it is clear that the critical velocity depends upon.

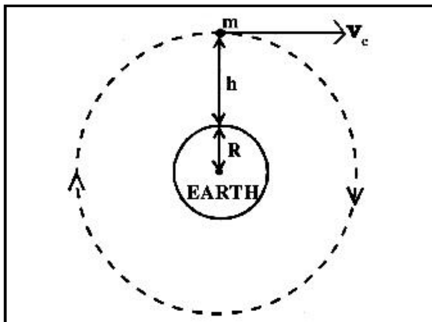
1. Mass of the earth
2. Radius of earth and
3. Height of the satellite above the surface of the earth.

7. PERIOD OF REVOLUTION OF A SATELLITE

The time taken by a satellite to complete one revolution round the earth is called its **period or periodic time** (T).

Consider a satellite of mass m revolving in a circular orbit $_c$ at a height h above the surface of the earth. Let M and R be the mass and the radius of the earth respectively. The radius (r) of the circular orbit of the satellite is r = R + h.

For the circular motion,



CIRCULAR ORBIT OF A SATELLITE

$$\therefore \quad v_c = \sqrt{\frac{GM}{r}} \qquad \text{...(i)}$$

If T is the period of revolution of the satellite,

$$\text{Period (T)} = \frac{\text{circumference of orbit}}{\text{critical velocity}} = \frac{2\pi r}{v_c}$$

$$T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} \qquad \text{...(From i)}$$

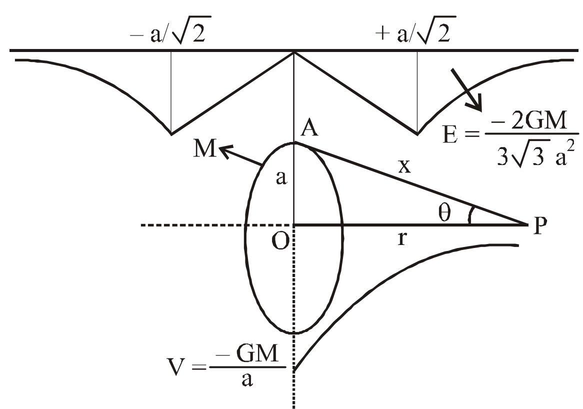
$$\therefore \quad T = 2\pi\sqrt{\frac{r^3}{GM}}$$

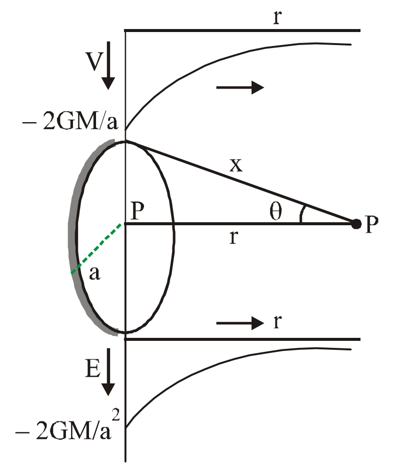
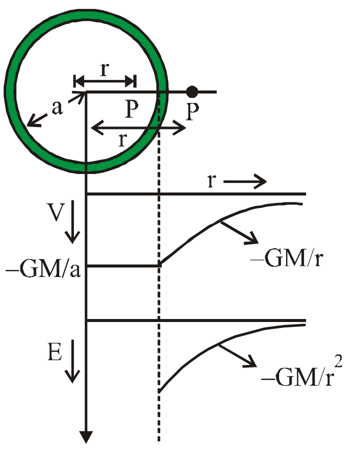
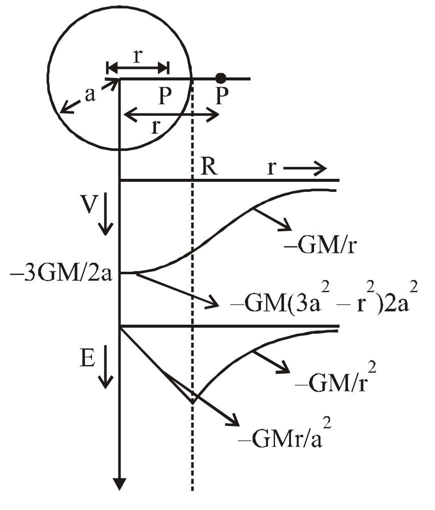
This expression gives the periodic time of the satellite. Squaring the expression, we get

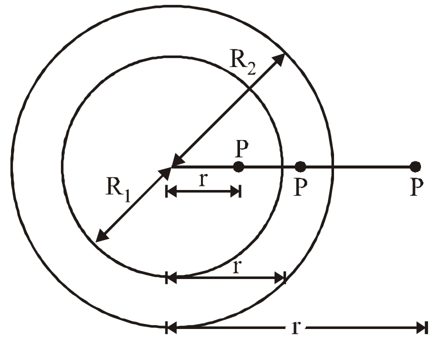
$$\therefore \quad T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\therefore \quad T^2 \propto r^3 \qquad \text{...(since G and M are constants)}$$

Thus, the square of the period of revolution of a satellite is directly proportional to the cube of the radius of its orbit.

Object	Potential (V)	Electric Field (E)		Figure
Ring	$V = \frac{-GM}{(a^2+r^2)^{1/2}}$	$\vec{E} = \frac{-GMr}{(a^2+r^2)^{3/2}}\hat{r}$		

Object	Potential (V)	Electric Field (E)	Figure
Thin Circular	$V = \frac{-2GM}{a^2}\left[\sqrt{a^2+r^2}-r\right]$	$\vec{E} = -\frac{2GM}{a^2}\left[1-\frac{r}{\sqrt{r^2+a^2}}\right]\hat{r}$	
Uniform Thin Spherical Shell (a) Point P inside the shell ($r \le a$) (b) Point P outside the shell ($r \ge a$)	$V = \frac{-GM}{a}$ $V = \frac{-GM}{r}$	$E = 0$ $\vec{E} = \frac{-GM}{r^2}\hat{r}$	
Uniform Solid Sphere (a) Point P inside the sphere ($r \le a$) (b) Point P outside the sphere ($r \ge a$)	$V = -\frac{Gm}{2a^3}(3a^2 - r^2)$ $V = -\frac{GM}{r}$	$\vec{E} = \frac{-GMr}{a^3}\hat{r}$ $\vec{E} = \frac{-GM}{r^2}\hat{r}$	

Object	Potential (V)	Electric Field (E)	Figure
Uniform Thick Spherical Shell			
(a) Point outside the shell	$V = -G\frac{M}{r}$	$\vec{E} = -G\frac{M}{r^2}\hat{r}$	
(b) Point inside the shell	$V = \frac{-3}{2}GM\left(\frac{R_2+R_1}{R_2^2+R_1R_2+R_1^2}\right)$	$\vec{E} = 0$	
(c) Point between the two surfaces	$V = \frac{-GM}{2r}\left(\frac{3rR_2^2-r^3-2R_1^3}{R_2^3-R_1^3}\right)$	$\vec{E} = \frac{-GM}{r^2}\left(\frac{r^3-R_1^3}{R_2^3-R_1^3}\right)\hat{r}$	

8. GRAVITATIONAL FIELD

The space surrounding any mass is called a gravitational feld. If any other mass is brought in this space, it is acted upon by a gravitational force. In short, the space in which any mass experiences a gravitational force, is called a **gravitational field.**

9. GRAVITATIONAL INTENSITY

The **gravitational intensity** at any point in a gravitational field is defined as the force acting on a unit mass placed at that point.

1. The gravitational intensity (E) at a point at distrance r from a point mass M is given by

$$E = \frac{GM}{r^2} \quad \text{(Where G is the constant of gravitation.)}$$

2. If a point mass m is placed in a gravitational field of intensity E, the force (F) acting on the mass m is given by

F = mE.

10. GRAVITATIONAL POTENTIAL

The **gravitational potential** at any point in a gravitational field is defined as the work done to bring a unit mass from infinity to that point.

1. The gravitational potential (V) at a point at distance r from a point mass M is given by,

$$V = -\frac{GM}{r} \quad \text{(Where G is the constant of gravitation)}$$

2. The work done on a unit mass is converted into its potential energy. Thus, the gravitational potential at any point is equal to the potential energy of a unit mass placed at that point.

3. If a small point mass m is placed in a gravitational field at a point where the gravitational potential is V, the gravitational potential energy (P.E.) of the mass m is given by.

P.E. = mass × gravitational potential

= mV

$$P.E. = -\frac{GMm}{r}$$

10.1 Gravitational Potential Energy

Gravitational potential energy of a body at a point is defined as the work done in bringing the body from infinity to that point.

Let a body of mass m is displaced through a distance 'dr' towards the mass M, then work done given by,

$$dW = F\,dr = \frac{GMm}{r^2}dr \Rightarrow \int dW = \int_{\infty}^{r}\frac{GMm}{r^2}dr$$

Gravitational potential energy, $U = -\frac{GMm}{r}$

(i) From above equation, it is clear that gravitational potential energy increases with increase in distance (r) (i.e. it becomes less negative).

(ii) Gravitational P.E. becomes maximum (or zero) at $r = \infty$.

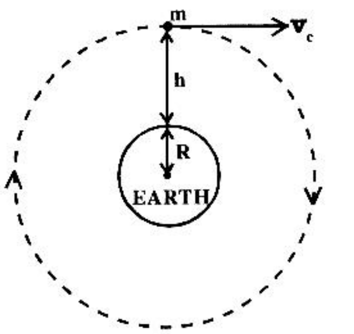
10.2 Expressions for different Energies of Satellite

1. Potential Energy
2. Kinetic Energy

3. Total Energy and
4. Binding energy

Let M = mass of the earth
R = radius of the earth
m = mass of the satellite
G = constant of gravitation
h = height of satellite

1. **Potential energy (P.E.) :** The satellite is at a distance (R + h) from the centre of the earth.



CIRCULAR ORBIT OF A SATELLITE

$$U = -\frac{Gm_1m_2}{r}$$

$$-\frac{GMm}{R+h} = U$$

2. **Kinetic energy (K.E.) :** The satellite is revolving in a circular orbit with the critical velocity (v_c). Hence its kinetic energy is given by,

$$K.E. = \frac{1}{2} m v_c^2$$

But $v_c = \sqrt{\frac{GM}{R+h}}$

$$\therefore \quad K.E. = \frac{1}{2} m\left(\frac{GM}{R+h}\right) = \frac{GMm}{2(R+h)}$$

3. **Total energy (T.E.)**

T. E = P.E. + K.E.

$$= -\frac{GMm}{R+h} + \frac{GMm}{2(R+h)} = -\frac{GMm}{2(R+h)}$$

The –ve sign indicates that the satellite is bound to the earth.

4. **Binding energy (B.E.) :** From the expression for the total energy, it is clear that if the satellite is given energy equal to $+\frac{GMm}{2(R+h)}$ the satellite will escape to infinity where its total energy is zero.

$$\therefore \quad B.E. = -(T.E.) = -\left[-\frac{GMm}{2(R+h)}\right] = +\frac{GMm}{2(R+h)}$$

5. **Binding Energy of a satellite**

The minimum energy which must be supplied to a satellite, so that it can escape from the earth's gravitation field, is called the **binding energy of a satellite.**

When the body of mass m is at rest on the earth's surface, its gravitational potential energy is given by,

$$U = -\frac{GMm}{R}$$

If the body is give an energy equal to $+\frac{GMm}{R}$, it will escape to infinity.

$$\therefore \quad \text{Binding energy of the body} = +\frac{GMm}{R}$$

11. ESCAPE VELOCITY OF A BODY

11.1 Expression for the escape velocity of a body at rest on the earth's surface

The minimum velocity with which a body should be projected from the surface of the earth, so that it escapes from the earth's gravitational field, is called the escape velocity. Thus, if a body or a satellite is given the escape velocity, its kinetic energy of projection will be equal to its binding energy.

Kinetic Energy of projection = Binding Energy.

$$\therefore \quad \frac{1}{2} mv_e^2 = \frac{GMm}{R}$$

$$\therefore \quad v_e = \sqrt{\frac{2GM}{R}}$$

11.2 Expression for 'V_e' in terms's of 'g'

The escape velocity for any object on the earth's surface is given by.

$$v_e = \sqrt{\frac{2GM}{R}}$$

If m is the mass of the object, its weight mg is equal to the gravitational force acting on it.

$$\therefore \quad mg = \frac{GMm}{R^2}$$

$$\therefore \quad G \quad M = gR^2$$

Substituting this value in the expression for v_e we get,

$$v_e = \sqrt{2gR}$$

11.3 Expression for the escape velocity of a body from Earth in terms of mean density of the planet

1. Derive expression for

$$v_e = \sqrt{\frac{2GM}{R}}$$

2. Let ρ be the mean density of the planet. Then,

$$M = \frac{4}{3}\pi R^3 \rho$$

$$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho}$$

$$v_e = 2R\sqrt{\frac{2\pi G\rho}{3}}$$

11.4 The escape velocity of a body from the surface of the earth is $\sqrt{2}$ times its critical velocity when it revolves close to the earth's surface

Let M and R be the mass and radius of the earth and m be the mass of the body. When orbiting close to the earth's surface, the radius of the orbit is almost equal to R. If v_c is the critical velocity of the body, then for a circular orbit.

Centripetal force = Gravitational force

$$\therefore \quad mv_c^2 = \frac{GMm}{R^2}$$

$$\therefore \quad v_c = \sqrt{\frac{GM}{R}} \qquad \text{...(i)}$$

If v_e is the escape velocity from the earth's surface,

K.E. of projection = Binding energy

$$\therefore \quad \frac{1}{2} mv_e^2 = \frac{GMm}{2}$$

$$\therefore \quad v_e = \sqrt{\frac{2GM}{R}} \qquad \text{...(ii)}$$

From Eq (i) and Eq. (ii), we get,

$$v_e = \sqrt{2}\, v_c$$

12. COMMUNICATION SATELLITE

An artificial satellite revolving in a circular obrit round the earth in the same sense of the rotational of the earth and having same period of revolution as the period of rotation of the earth (i.e. 1 day = 24 hours = 86400 seconds) is called as geo-stationary or communication satellite.

As relative velocity of the satellite with respective to the earth is zero it appears stationary from the earth's surface. Therefore it is know as geo-stationary satellite or geosynchronous satellite.

1. The height of the communication satellite above the earth's surface is about 36000 km and its period of revolution is 24 hours or 24 × 60 × 60 seconds.
2. The satellite appears to be at rest, because its speed relative to the earth is zero, hence it is called as geostationary or geosynchronous satellite.

12.1 Uses of the communication satellite

1. For sending TV signals over large distances on the earth's surface.
2. Telecommunication.
3. Weather forescasting.
4. For taking photographs of astronomical objects.
5. For studying of solar and cosmic radations.

13. WEIGHTLESSNESS

1. The gravitational force with which a body is attracted towards the centre of earth is called the weight of body. Weightlessness is a moving satellite is a feeling. It is not due to weight equal to zero.
2. When an astronaut is on the surface of earth, gravitational force acts on him. This gravitational force is the weight of astronaut and astronant exerts this force on the surface of earth. The surface of earth exerts an equal and opposte reaction and due to this reaction he feels his weight on the earth.
3. for an astronaut in an orbiting satellite, the satellite and astronaut both have same acceleration towards the centre

of earth and this acceleration is equal to the acceleration due to gravity of earth.

4. Therefore astronaunt does not produce any action on the floor of the satellite. Naturally the floor does not exert any force of reaction on the astronaut. As there is no reaction, the astronaut has a feeling of weightlessness. (i.e. no sense of his own weight).

Note...

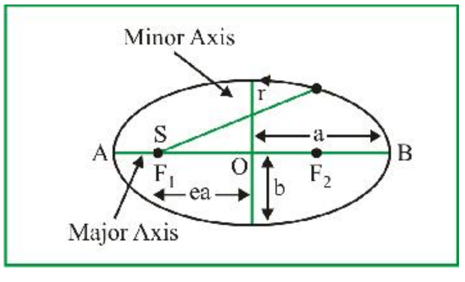
1. The sensation of weightlessness experienced by an aastronaut is not the result of there being zero gravitational acceleration, but of there being zero difference between the acceleration of the spacecraft and the acceleration of the astronaut.
2. The most common problem experienced by astronauts in the initial hours of weightlessness is known as space adaptation snydrome (space sickness).

14. KEPLER'S LAWS

14.1 Law of Orbit

Each Planet move surround the sun in an elliptical orbit with the sun at one of the foci as shown in figure. The eccentricity of an ellipse is defined as the ratio of the distance SO and AO i.e. $e = \frac{SO}{AO}$

$\therefore \quad e = \frac{SO}{a} \quad SO = ea$



The distance of closest approach with sun at F_1 is AS. This distance is called perigee. The greatest distance (BS) of the planet from the sun is called apogee.

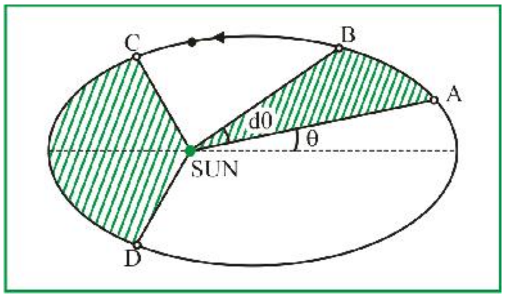
Perigee (AS) = AO – OS = a – ea = a (1 – e)

apogee (BS) = OB + OS = a + ea = a (1 + e)

14.2 Law of Area

The line joining the sun and a planet sweeps out equal areas in equal intervals of time. A planet takes the same time to travel from A to B as from C to D as shown in figure.

(The shaded areas are equal). Naturally the planet has to move faster from C to D.



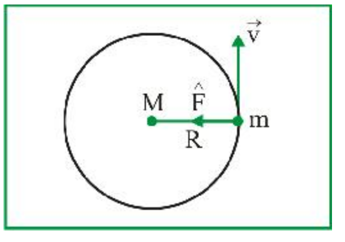
$$\text{Areal velocity} = \frac{\text{area swept}}{\text{time}}$$

$$= \frac{\frac{1}{2}r(rd\theta)}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \text{cosntant}$$

Hence $\frac{1}{2}r^2\omega = \text{constant}.$

14.3 Law of Periods

The square of the time for the planet to complete a revolution about the sun is proportional to the cube of semimajor axis of the elliptical orbit.



i.e. Centripetal force = Gravitational force

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow \frac{GM}{R} = v^2$$

Now, velocity of the planet is

$$v = \frac{\text{Circumference of the circular orbit}}{\text{Time period}} = \frac{2\pi R}{T}$$

Substituting Value in above equation

$$\Rightarrow \frac{GM}{R} = \frac{4\pi^2R^2}{T^2} \quad \text{or} \quad T^2 = \frac{4\pi^2R^3}{GM}$$

Since $\left(\frac{4\pi^2}{GM}\right)$ is constant,

$$\therefore T^2 \propto R^3 \quad \text{or} \quad \frac{T^2}{R^3} = \text{constant}$$

14.4 Gravity

Gravity is the force of attraction exerted by earth towards is centre on a body lying on or near the surface of earth. Gravity is merely a special case of gravitation and is also called earth's gravitational pull.

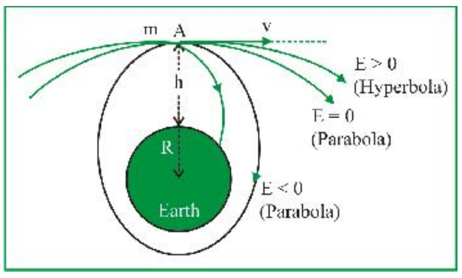
Weight of a body is defined as the force of attraction exerted by the earth on the body towards its centre.

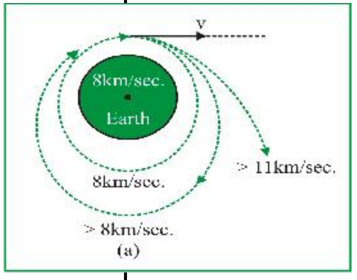
The units and dimensions of gravity pull or weight are the same as those of force.

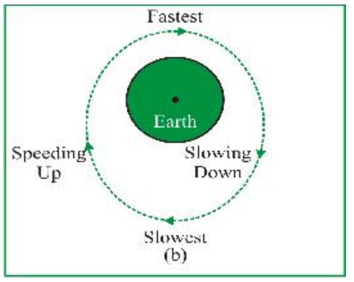
Astronomical Data

Body	Sun	Earth	Moon
Mean radius, m	6.95×10^{8}	6.37×10^{6}	1.74×10^{6}
Mass, kg	1.97×10^{30}	5.96×10^{24}	7.30×10^{22}
Mean density, 10^3 kg/m^3	1.41	5.52	3.30
Period of rotation about axis, days	25.4	1.00	27.3

LAUNCHING OF AN ARTIFICIAL SATELLITE AROUND EARTH







The satellite is placed upon the rocket which is launched from the earth. After the rocket reaches its maximum vertical height h, a spherical mechanism gives a thrust to the satellite at point A (figure) producing a horizontal velocity v. The total energy of the satellite at A is thus,

$$E = \frac{1}{2} mv^2 - \frac{GMm}{R+h}$$

The orbit will be an ellipse (closed path), a parabola, or an hyperbola depending on whether E is negative, zero, or positive. In all cases the centre of the earth is at one focus of the path. If the energy is too low, the elliptical orbit will intersect the earth and the satellite will fall back. Otherwise it will keep moving in a closed orbit, or will escape from the earth, depending on the values of v and R.

Hence a satellite carried to a height h (<< R) and given a horizontal velocity of 8 km/sec will be placed almost in a circular orbit around the earth (figure). If launched at less than 8 km/sec, it would get closer and closer to earth until it hits the ground. Thus 8 km/sec is the critical (minimum) velocity.

14.5 Intertial mass

Inertial mass of a body is related to its inertia in linear motion; and is defined by Newton's second law of motion.

Let a body of mass m_i move with acceleration a under the action of an external force F. According to Newton's second law of motion, $F = m_i a$ or $m_i = F/a$

Thus, inertial mass of a body is equal to the magnitude of external force required to produce unit acceleration in the body.

14.6 Gravitational mass

Gravitational mass of a body is related to gravitational pull on the body, and is defined by Newton's law of gravitational.

$$F = \frac{GM m_G}{R^2} \quad \text{or} \quad m_G = \frac{F}{\left(GM/R^2\right)} = \frac{F}{I}$$

The mass m_G of the body in this sence is the gravitational mass of the body. The intertia of the body has no effect on the gravitational mass of the body.

$m_G = F$

Thus, **Gravitational mass** of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.

14.7 Centre of Gravity

Centre of gravity of a body placed in the gravitational field is that point where the net gravitational force of the field acts.